FRG APPROACH TO NUCLEAR MATTER IN EXTREME CONDITIONS

HARMONIC EXPANSION OF THE EFFECTIVE POTENTIAL AT FINITE CHEMICAL POTENTIAL

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INTRODUCTION TO THE FUNCTIONAL RENORMALIZATION GROUP METHOD (FRG)

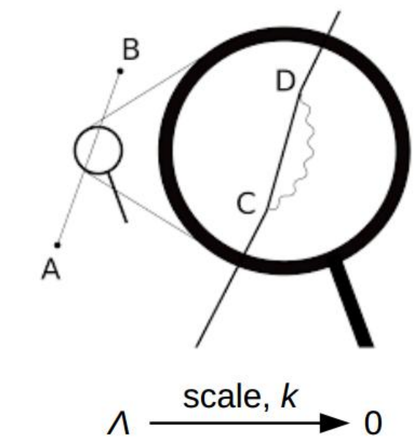
Calculating thermodynamic quantities in interacting quantum field theories on **finite chemical potential** is challenging. Functional renormalization group (**FRG**) is a general method to calculate the **effective action** of the system. Using a renormalization scale the FRG method can provide an effective action that incorporates **quantum fluctuations**. Thermodynamic quantities can be derived from this effective action. The scale dependence of the effective action (Γ_k) is determined by the **Wetterich-equation**, which is an **exact** equation if the form of effective potential is given. The theory is defined at some UV scale, by the UV-scale effective action. Using this as the initial condition we integrate the Wetterich equation to determine the effective action at *k=0.* This way the effective action at *k=0,* contains all quantum mechanical effects.

FRG:

- Exact solution
- Non-perturbative description
- Continous transition from microscopic to macroscopic scale

$$\partial_k \Gamma_k = \frac{1}{2}\int dp^D\, STr\left[\frac{\partial_k R_k}{\Gamma_k^{(1,1)} + R_k}\right]$$

$\Gamma_{k=\Lambda}$ UV scale, classical ⟹ $\Gamma_{k=0}$ IR scale, included quantum fluctuations



INTERACTING FERMI-GAS MODEL

- Simple model of massless fermions which are coupled to the self interacting bosons using Yukawa-coupling
- Can be used to study the behavior of the Wetterich-equaiton at zero temperature and finite chemical potential
- Can be used to find a general method to solve FRG equations for fermions on finite chemical potential and zero temperature
- Easily extended to include vector mesons, hence it can be used as a generalization of the *σ-ω model (Walecka-model)*

$$\Gamma_k[\varphi,\psi] = \int d^4x\left[\bar{\psi}\left(i\partial\!\!\!/ - g\varphi\right)\psi + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - U_k(\varphi)\right]$$

Fermions : *m=0,* **Yukawa-coupling** generates mass for fermions

Bosons: the **potential** contains self interaction terms

We study the scale dependence of the potential only!!

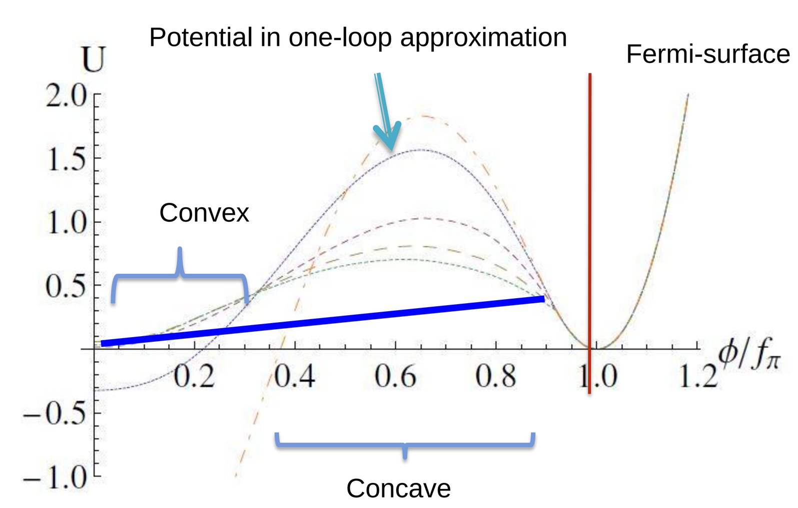
The Wetterich-equation for the Fermi-gas model:

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\underbrace{\frac{1+2n_B(\omega_B)}{\omega_B}}_{\text{Bosonic part}} + 4\underbrace{\frac{-1+n_F(\omega_F-\mu)+n_F(\omega_F+\mu)}{\omega_F}}_{\text{Fermionic part}}\right]$$

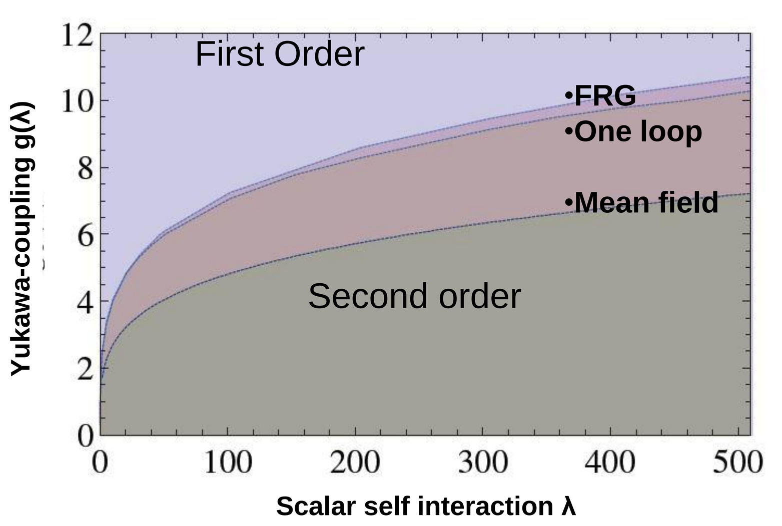
$$U_\Lambda(\varphi) = \frac{m_0^2}{2}\varphi^2 + \frac{\lambda_0}{24}\varphi^4 \qquad \omega_F^2 = k^2 + g^2\varphi^2 \qquad \omega_B^2 = k^2 + \partial_\varphi^2 U \qquad n_{B/F}(\omega) = \frac{1}{1\mp e^{-\beta\omega}}$$

- The differential equation has to be solved at zero temperature and finite chemical potential to provide an equation of state of nuclear matter in neutron stars.
- The fermionic and bosonic degrees of freedom are separated into two terms
- The *Yukawa-coupling (g)* modifies the the chemical potential of the fermions
- At zero temperature the Fermi-distribution becomes a step funciton which makes this equation difficult to solve, because the lack of stability of the result

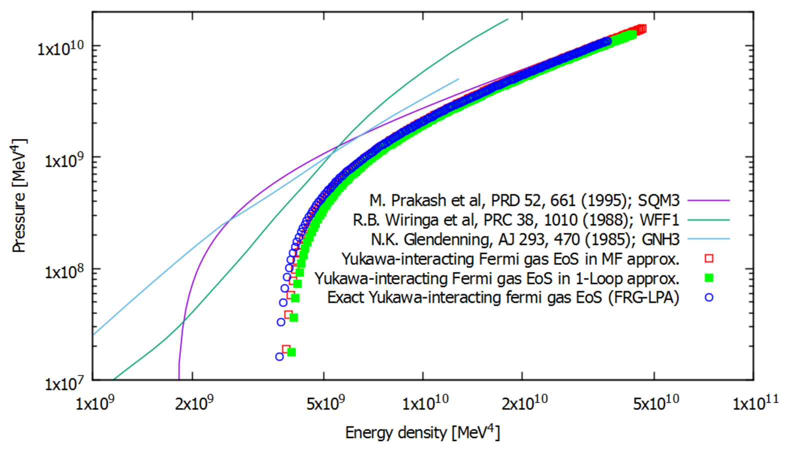
RESULTS



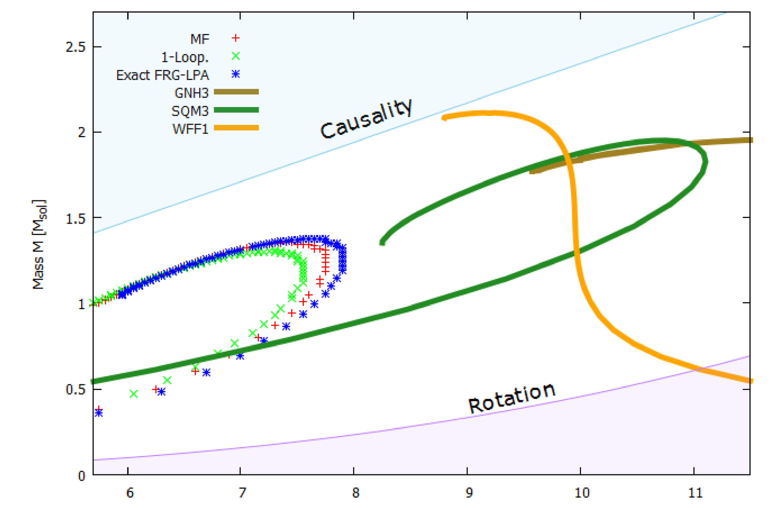
- The solution converges fast as the order of the expansion increases where the potential is **convex.**
- Where the potential is **concave** the solution converges slowly to a line, because the IR potential can never be concave: this is the **Maxwell-construction.**
- Using this knowledge it is enough to compute the potential until the convex parts converge: This is the **Coarse Grained IR effective action**, which can be used to calculate thermodynamics.



- The **phase diagram** of the interacting Fermi gas in different approximations shows the type of the **phase transition** in the system when the **couplings** are specified.
- The FRG and **one loop calculations are very** close and both of them are different from the main field result: **FRG is a relevant** improvement. (quantum effects)
- **FRG makes the phase transition smoother**: first order in mean field turns into second order in the FRG calculation.



- Comparison of the interacting Fermi gas EOS in different approximations to other models. (SQM3, WFF1, GNH3)
- **Consistency**: at high energies they approach SQM3.
- The mean field, one loop and exact calculations gives very **similar equation of state**, despite their different results in the phase diagram.



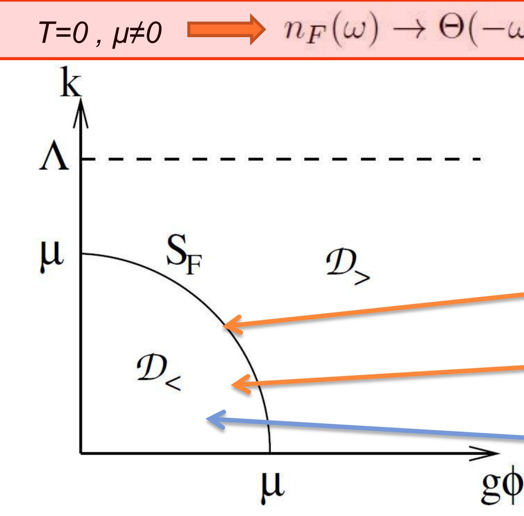
- The **M-R diagram** corresponds to the equation of state as above.
- The one-loop calculation was close to the exact result in the case of the phase diagram, but in the M-R diagram the **Mean field calculation is better approximation**.
- The **maximum mass of the stable neutron star is increased** in the case of the exact solution compared to the one loop and mean field approximations. M < 1,5 M_{sol} and R < 8 km

SOLUTION METHOD

At zero temperature, the Fermi-Dirac distribution becomes a step function and divides the *k-φ plane* into two different regions. There is a differential equation corresponding to each region which has to be solved separately, but they have to match at the boundary.

T=0 , μ≠0 ⟹ $n_F(\omega) \to \Theta(-\omega)$

We have two equations for the two values of the step function each valid on different domain



$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B} - \frac{4}{\omega_F}\right]$$

$k_F = \sqrt{\mu^2 - g^2\varphi^2}$, Fermi-surface

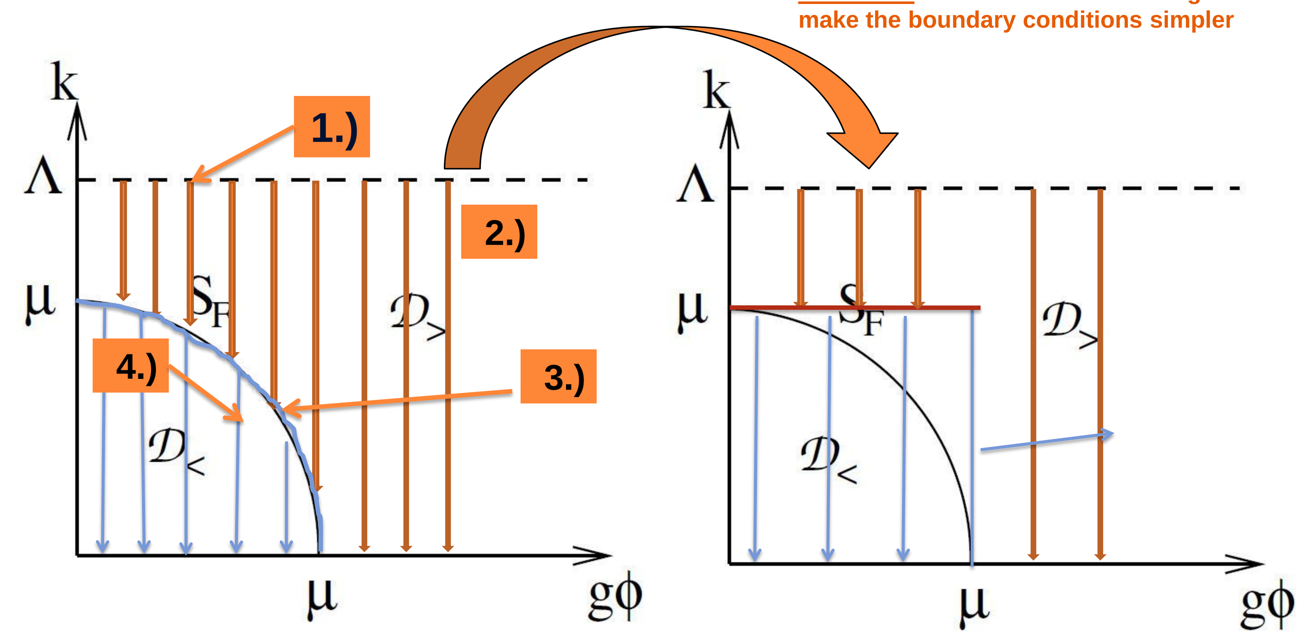
$$\partial_k U_k = \frac{k^4}{12\pi^2}\frac{1}{\omega_B}$$

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel

Solution in steps

Transform the circle into a rectangle to make the boundary conditions simpler

1) Fix the high scale couplings in the theory at scale Λ.
2) Integrate the equation which is valid outside of the Fermi surface.
3) Calculate the initial conditions for the equation inside the Fermi surface.
4) Integrate the equation which is valid below the Fermi surface.



Orthognal system: harmonic functions

The solution is expanded in an **orthonormal function basis** which matches the transformed boundary conditions. The square root is expanded to study the behavior of the approximation. The transformed equation with the expansions:

$$xc_n'(x) = \int_0^1 dy\, h_n(y)\left[-xV_0' + y\partial_y\tilde{u} - \underbrace{\frac{g^2(kx)^3}{12\pi^2}\sum_{p=0}^{\infty}\binom{-1/2}{p}\frac{(\partial_y^2\tilde{u} - M^2)^p}{\omega^{2p+1}}}_{\text{Expanded squre root}}\right]$$

$$\tilde{u}(x,y) = \sum_{n=0}^{\infty} c_n(x)h_n(y) \qquad h_n(1) = 0 \qquad \int_0^1 dy\, h_n(y)h_m(y) = \delta_{nm} \qquad h_n(y) = \sqrt{2}\cos q_n y,$$

The Expansion is necessary to accomodate the sharp boundary condition, which is forced on the differential equation by the Fermi surface on zero temperature and the finite chemical potential.

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[1] A. Jakovác, G. Barnaföldi and P. Pósfay PoS(EPS-HEP2015) 369

[2] A. Jakovác, A. Patkós and P. Pósfay, Eur. Phys. J C75:2

[3] A. Jakovác, G. Barnaföldi and P. Pósfay The proceedings of SQM 2016

TAKE-HOME MESSAGE

1. **We find a general method to solve the FRG equations at finite chemical potential and zero temperature**
2. **This opens the way to calculate thermodynamics and phase structure for effective QCD models at the low temperature and high denisty part of the QCD phase diagram, which might appears in neutron stars.**

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