FUNCTIONAL RENORMALIZATION GROUP FOR FERMIONIC FIELDS

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INTRODUCTION TO THE FUNCTIONAL RENORMALIZATION GROUP METHOD (FRG)

The basic principle: computation of *quantum n-point correlation functions* by gradual path integration. To achieve this, the generator functional is modified with a kinetic term, called the Regulator. The Regulator is constructed to suppress modes below a certain scale: *k*. The **scale dependent effective action,** witch is the generator of the connected Feynmann-diagrams, is calculated by the Legendre-transformaiton of the Schwinger-functional.

$$Z_k[J] = \int \left(\prod_a d\Psi_a\right) e^{-S[\Psi]-\frac{1}{2}R_{k,ab}\Psi_a\Psi_b+\Psi_a J_a}$$

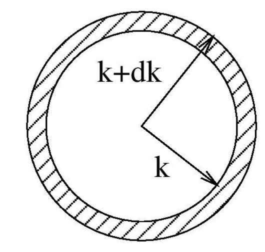
Legendre-transformation →

$$\Gamma_k[\psi] = \sup_J \left(\psi_a J_a - W[J]\right) - \frac{1}{2}R_{k,ab}\psi_a\psi_b$$

The Scale dependence of the effective action is determined by the **Wetterich-equation**, witch is an exact equation if the form of effective potential is given. The theory is defined at some UV scale, by the UV-scale effective action. Using this as the initial condition we integrate the Wetterich equation to determine the effective action at *k=0.* This way the effective action at *k=0,* contains all quantum mechanical effects.

$$\partial_k \Gamma_k = \frac{1}{2} Str\left[\left(\partial_k R_k\right)\left(\Gamma_k^{(1,1)} + R_k\right)^{-1}\right]$$

***k* starts at UV scale: classical** → ***k=0* : included quantum effects**



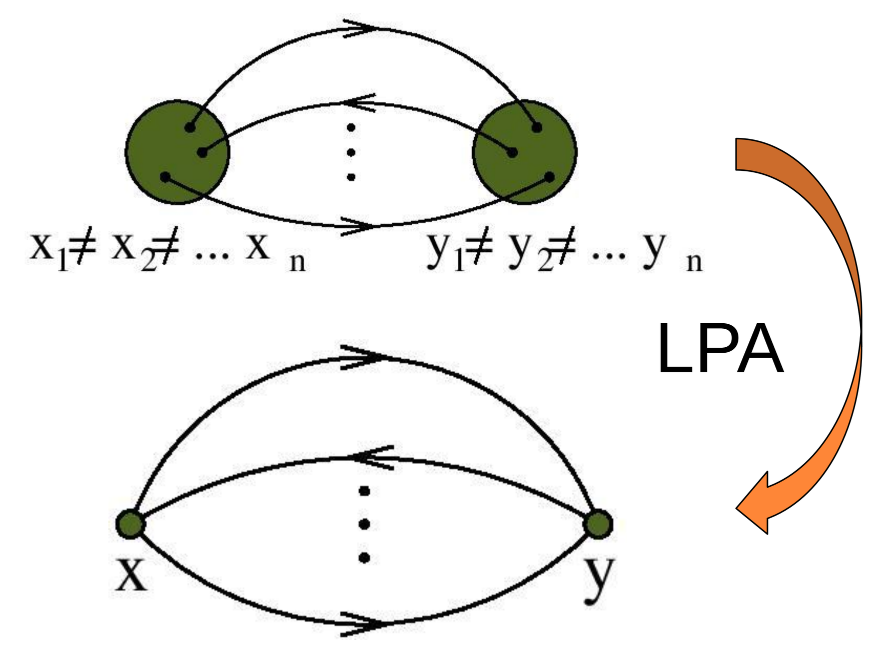
LOCAL POTENTIAL APPROXIMATION (LPA)

In the theory we should include all operators, in the UV effective action, even the most exotic ones, because they might become relevant at lower scale values. Since this task is impossible to do, in practice an ansatz is neded for the effective action.

The local potential approximation, is based on the assumption that propagators vary in spacetime much slower than vertices. This implies that the UV-scale effective action has the following form:

$$\Gamma_k[\psi] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k(\psi)\right]$$

The effective action consists of a kinetic term with some kernel *K*, and a potential, *U* witch depends on the invariant combinations of the fields. This way Wetterich-equation becomes a differntial equation for the potential.



Any invariant combination of the fields can appear in the potential, on arbitrary high power, even for fermionic combinations!

Local potential approximation is based on the assumption that the value of the two diagrams are close. Vertex of the upper diagram is:

$$\Gamma_k^{(n)}(x_1,\dots,x_n)\Psi(x_1)\dots\Psi(x_{2n})$$

The vertex of the lower diagram:

$$U_k^{(n)} \lim_{\Delta V\to 0}\left(\frac{1}{\Delta V}\int_{\Delta V}\bar\psi(x)\psi(x)\right)^n \xrightarrow{\text{notation}} U_k^{(n)}(\bar\psi(x)\psi(x))^n$$

This means that, the fields have different cooridante variables, but it is a good approximation to take their value at a common cooridante, witch charaterizes the vertex on the upper diagram. This can be interpreted as some compositness scale: above this scale the vertex appears as a point, below this scale we can think of it as a patch of very close points.

GROSS-NEVEU MODEL

The Gross-Neveu model is given by the following effective action:

$$\Gamma[\bar\psi,\psi] = \int d^dx \left\{\sum_{j=1}^{N_f}\bar\psi_j \partial\!\!\!/ \psi_j + U\left[\left(\sum_{j=1}^{N_f}\bar\psi_j\psi_j\right)^2\right]\right\}$$

The model has N_f number fermionic fields, and can be treated with the method of partial bosonisation. Due tothe previously mentioned properties of the local potential approximation, it can be solved with LPA method. It is useful to write the Wetterich-equation of the model with dimensionless rescaled variables:

$$\partial_t y = -dy + 2(d-1)xy' - \frac{1}{1+4y'^2x} - \frac{1}{4N_f}\left[\frac{1}{1+4y'^2x} - \frac{1}{1+12y'^2x+16y'y''x^2}\right]$$

Where:

$$u_k = k^{-d}U_k \qquad i_{GN} = k^{2(1-d)}I_{GN} \qquad x = (4Q_dN_f)^{-2}i_{GN}$$

$$I_{GN} = \left(\bar\psi\psi\right)^2 \qquad y = (4Q_dN_f)^{-1}u_k$$

In order to solve this differential equation the potential has to be truncated:

$$y_k(x) = \sum_{n=1}^{n_{max}} \frac{l_{n,k}}{n}x^n$$

If we substitute this truncation back to the Wetterich-equation, it yields a tower of coupled differential equations for the scale-dependendent coupling constants. By solving these differnetial equations, we can determine the scale dependence of the coupling constants and consequently the effective action.

We interested in the points, where the running of the coupling constants stops. These points called fixed points, and their formal definition are:

$$\left.\begin{aligned}\partial_t\Gamma_k[\phi] &= \sum_i \beta_{i,k}O_i\\ \beta_{i,k} &= \partial_t g_{i,k}\end{aligned}\right\}\quad \beta_{i,k} = 0\ \forall i.$$

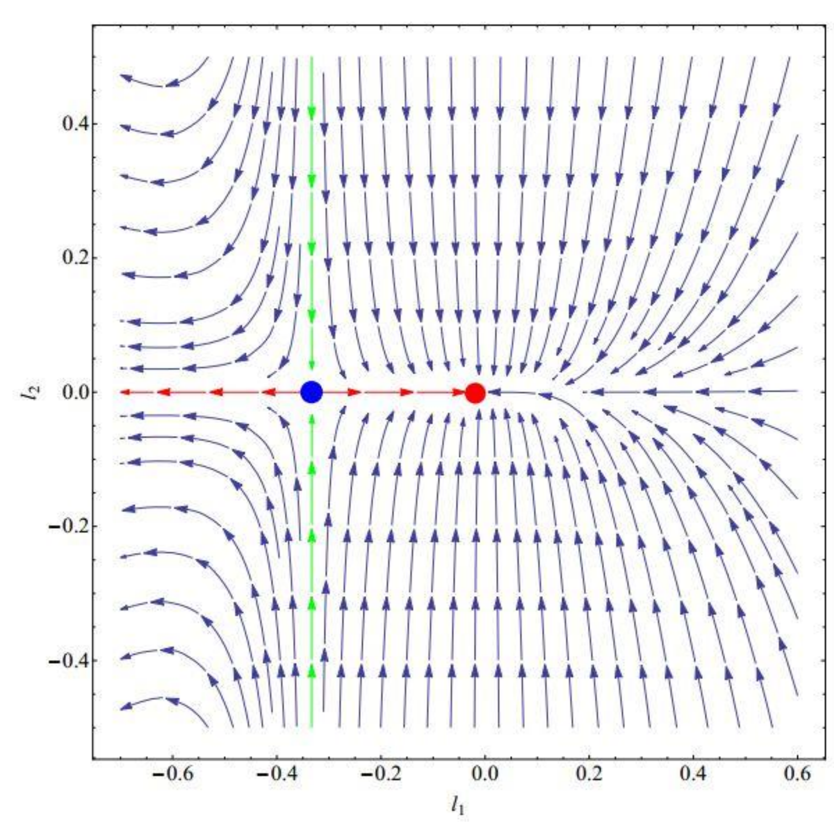
Where O_i are the operators in the effective action, and β_i are the beta-functions, witch are the scale-derivatives of the running coupling constants. When the beta-functions are zero the coupling constants do not changes with the scale. Every theory has a fixpoint, where all coupling constans are zero. This type of fixed points called Gaussian fixed point. If a theory has a non-Gaussian fixed point, it can be extended to arbitrary high scale because the coupling constatns must be finite.

RESULTS

There are two fixed points in the Gross-Neveu model: a Gaussian and a non-Gaussian one. The fixpoint structure of the theory is shown on the right figure, in the case of second order truncation. We can see the running of the two coupling constants: every points in the diagram corresponds to one set of the coupling constants values. The arrows show, how the values of the coupling constants changes if we start to integrate the Wetterich-equation from some set of starting values.

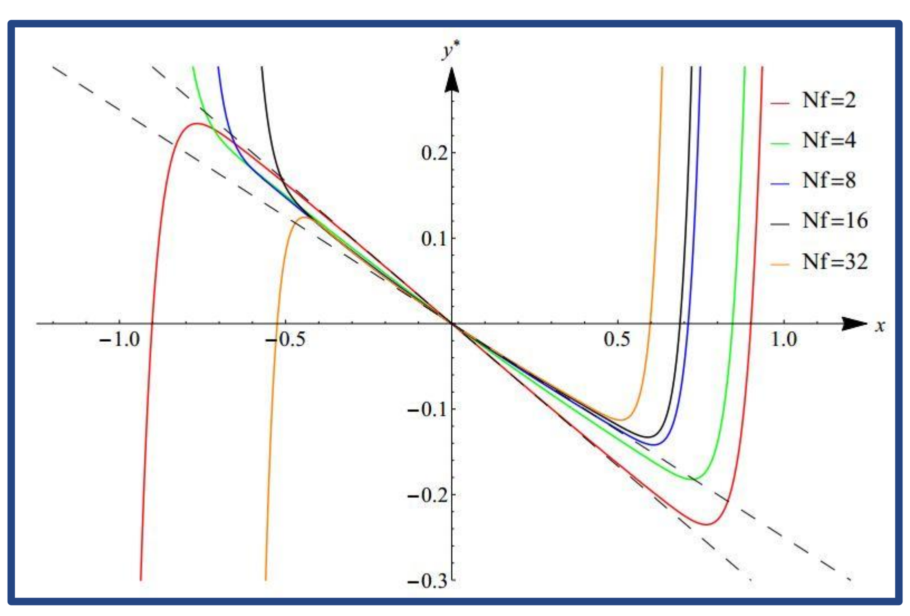
The fixpoint structure does not change when we increse the order of truncation, witch means that higher order coupling constants do not have any non-trivial fixed points. The coordinates of the fixed points are.:

- **Gaussian: l_1=0,l_2=0,…**
- **Non-Gaussian: l_1=-0.3,l_2=0,…**

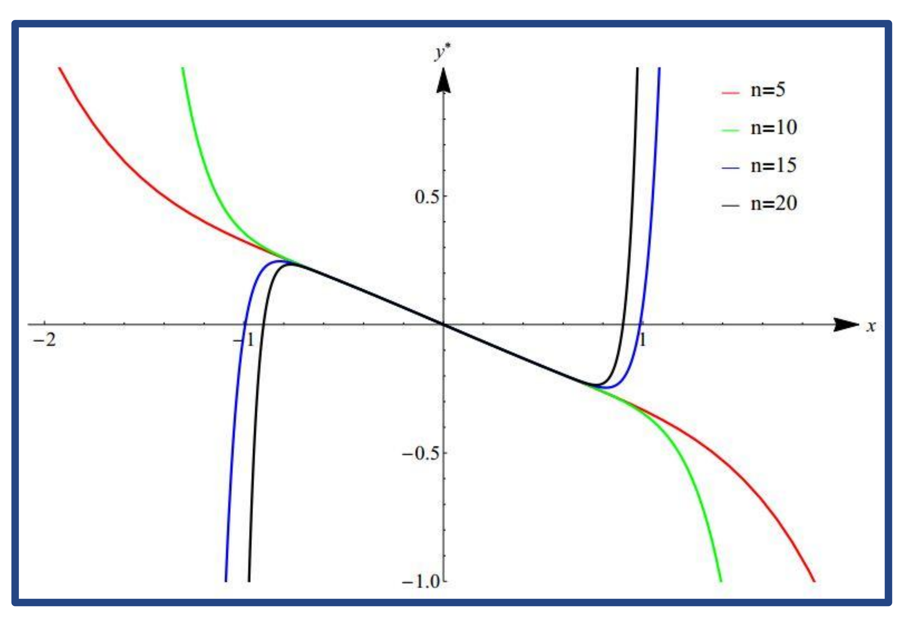


The effective potential at the fixed points is shown on the figures below. We can see that, as growing the number of fermionic fields, the fix point potential approximate the solution witch belongs to the infinte number of fields (dashed line).

These results are consistent with the result of partial bosonisation method, and presents that it is indeed possible to use high order truncations in LPA approximations of fermionic systems.



Fixed point potential at the non-Gaussian fixed point



Fixed point potential at the Gaussian fixed point

BIBLIOGRAPHY

[1] A. Patkos, Mod.Phys.Lett. A27 (2012) 1250212
[2] A. Jakovác and A. Patkós, Phys. Rev. D88, 065008 (2013)
[3] A. Jakovác, A. Patkós and P. Pósfay, Eur. Phys. J C75:2 (2015)

APPLICATIONS

- Fundamental matter is fermionic, Bose-condensation with fermionic bound states
- Higgs-mechanism,composite Higgs-models
- Scale dependent equation of state for nuclear matter, phase stransition points
- Neutron star stability tests

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