FRG Approach to Nuclear Matter in Extreme Conditions

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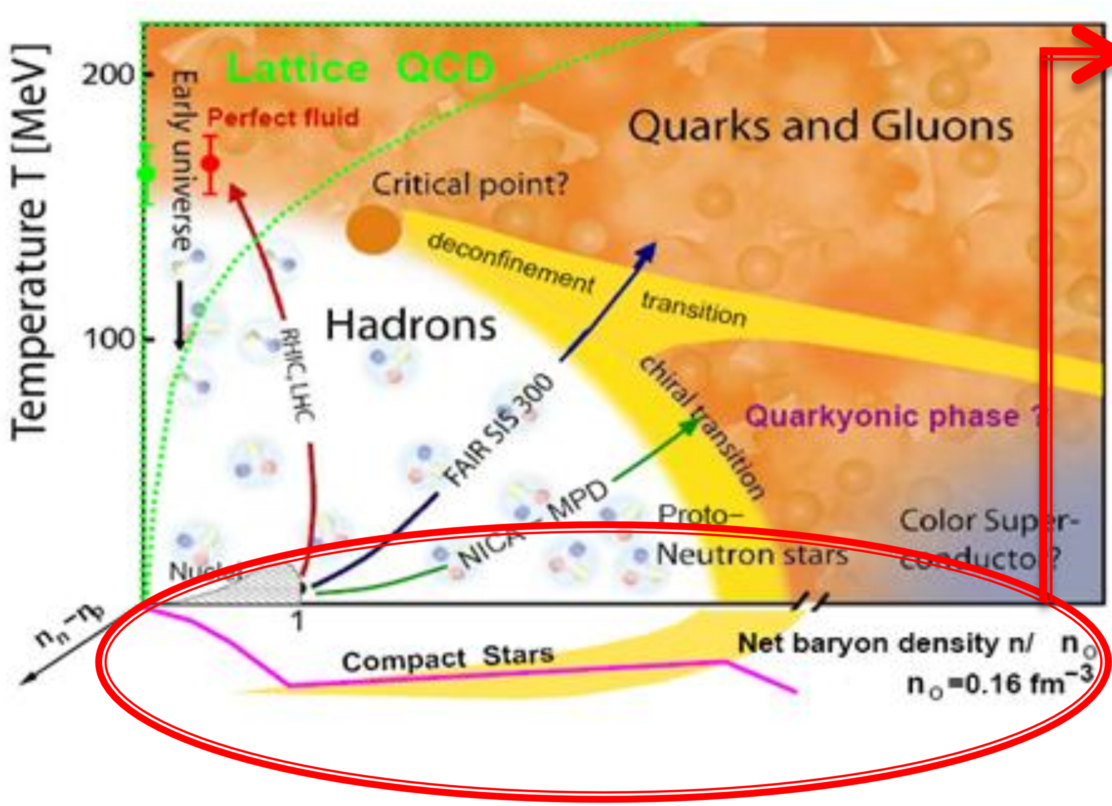
Outline

1. Motivation
2. Introduction to FRG
3. Solving the Wetterich-equation at finite chemical potential
4. Comparison between FRG results and other methods
5. Proof of concept: application for compact stars

QCD phase diagram

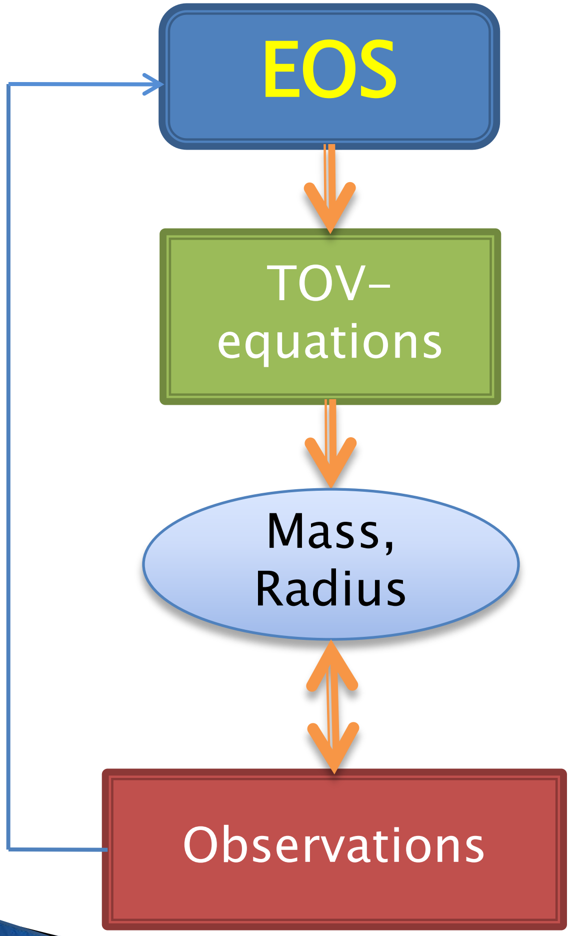


Hard to study area:

- ❑ no numerical results from lattice
- ❑ hard to reach with experiments

- Effective models
- indirect methods
- Astrophysics
- Compact stars

EoS of nuclear matter and compact stars



What are the effects of quantum fluctuations on the Equaiton of State (EOS) ?

What is the difference between the same parameters in mean field and quantum flucuations included ?

- Compressibility (important for neutron star mass!)
- Binding energy
- Surface tension of nuclear matter

FRG is a general method to take quantum fluctuations into account.

FAQ

Frequently asked questions

- **Why use renormalization in an effective theory?**

  Renormalization takes into account **quantum fluctuations.** This provides features one can not have in a mean field model.

- **What are these features?**
  - Quantum fluctuations play huge role in **phase transitions** – better description of phase transitions.
  - FRG has a built in **thermodynamical stability,** which is not present in many mean field constructions; for example: Walecka-model (the free energy is always convex)
  - Better consistency with the **quantum mechanic** nature of the particles.

FAQ

Frequently asked questions

- **What is the meaning of FRG in an effective theory?**

  It is a **cutoff theory.** It should provide a low energy effective description of **QCD** OR *Thinking in reverse: starting from low energy it could give us a hint of QCD at the cutoff: we can test what operators are important at that scale using the observations as constraints.*

- **Is the effect of quantum fluctuations relevant in the case of compact stars?**
  - It can change the **neutron star mass** for a given model, because the pressure of quantum fluctuations is taken into account
  - Possible new measurements (gravity waves) are more sensitive to the **phase structure**, which is better described using quantum fluctuations
  - **Masquarade problem**: many different model gives similar neutron star properties. Using FRG the quantum mechanical and thermodynamical **consistency** can help deciding between models.

Functional Renormalization Group (FRG)

- General non-perturbative method to determine the effective action of a system.
  - **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \; STr \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Wetterich equation

k=Λ
Classical action

Integration

k=0
Quantum fluctuations included

Functional Renormalization Group (FRG)

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$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \; STr \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Need an ansatz for the integration

Not necessarly perturbative ansatz!

$$\Gamma_k = \sum_{l=1}^{l=N} \frac{g_l(k)}{l!} \hat{O}_l$$

Scale dependent coupling

Functional Renormalization Group (FRG)

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$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \; STr \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Regulator:
- determines the modes present on scale *k*
- physics is regulator independent

Interacting Fermi-gas model

Ansatz for the effective action:

$$\Gamma_k[\varphi,\psi] = \int d^4x \left[\bar{\psi}\left(i\partial\!\!\!/ - g\varphi\right)\psi + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - U_k(\varphi)\right]$$

Fermions : *m=0*, **Yukawa-coupling** generates mass

Bosons: the **potential** contains self interaction terms

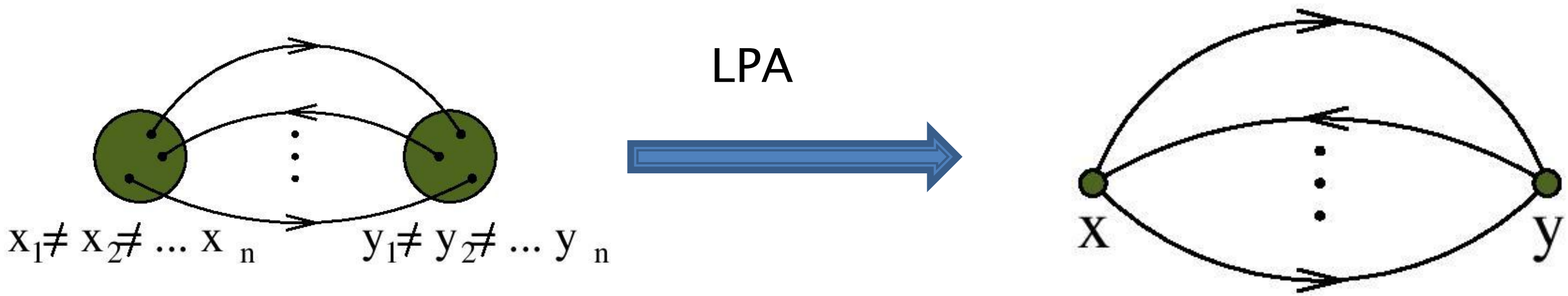
We study the scale dependence of the potential only!!

Local Potential Approximation (LPA)

What does the ansatz exactly mean ?

LPA is based on the assumption that the contribution of these two diagrams are close.

(momentum dependence of the vertices is suppressed)



This implies the following ansatz for the effective action:

$$\Gamma_k[\psi] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k(\psi)\right]$$

Interacting Fermi-gas at finite temperature

Ansatz for the effective action:

$$\Gamma_k[\varphi,\psi]=\int d^4x\left[\bar{\psi}\left(i\partial\!\!\!/-g\varphi\right)\psi+\frac{1}{2}\left(\partial_\mu\varphi\right)^2-U_k(\varphi)\right]$$

⇓ Wetterich -equation

$$\partial_k U_k=\frac{k^4}{12\pi^2}\left[\underbrace{\frac{1+2n_B(\omega_B)}{\omega_B}}_{\text{Bosonic part}}+4\underbrace{\frac{-1+n_F(\omega_F-\mu)+n_F(\omega_F+\mu)}{\omega_F}}_{\text{Fermionic part}}\right]$$

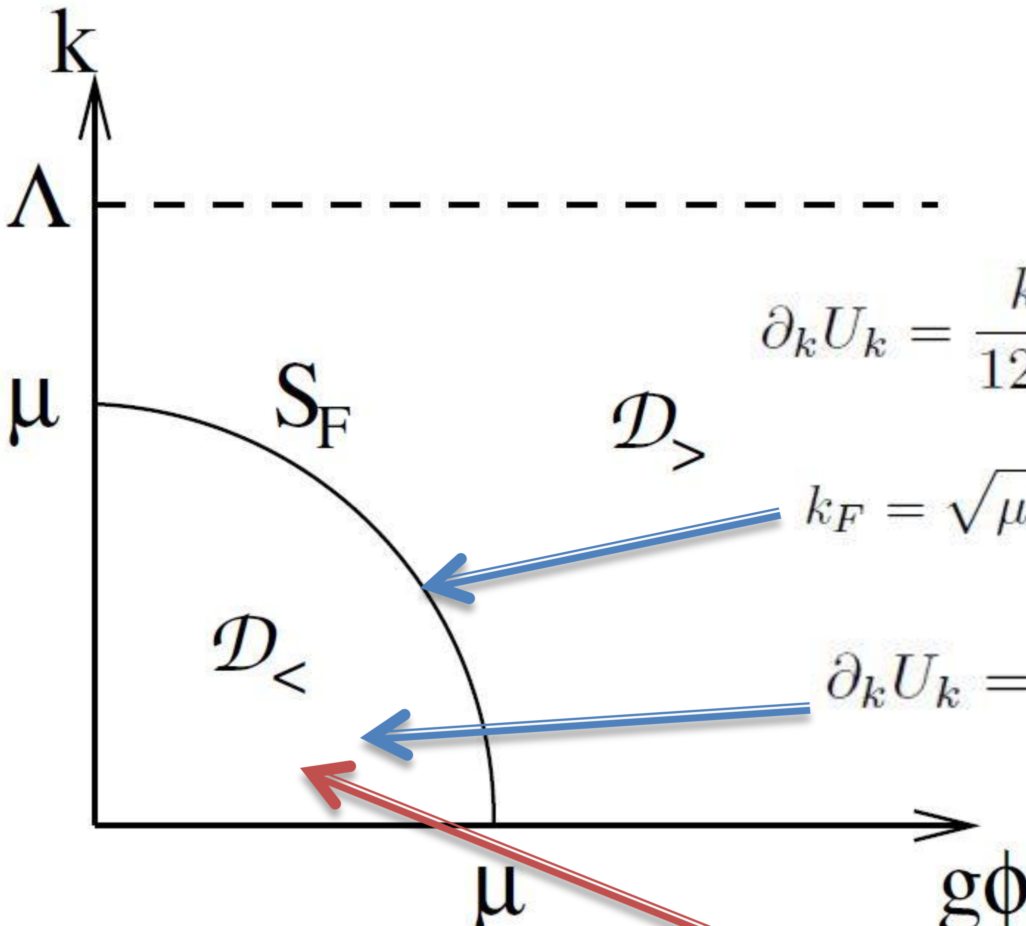
Bosonic part **Fermionic part**

$$U_\Lambda(\varphi)=\frac{m_0^2}{2}\varphi^2+\frac{\lambda_0}{24}\varphi^4 \qquad \omega_F^2=k^2+g^2\varphi^2 \qquad \omega_B^2=k^2+\partial_\varphi^2 U \qquad n_{B/F}(\omega)=\frac{1}{1\mp e^{-\beta\omega}}$$

Interacting Fermi-gas at zero temperature

$T=0\,,\ \mu\neq 0 \Rightarrow n_F(\omega)\to\Theta(-\omega)$

We have two equations for the two values of the step function each valid on different domain



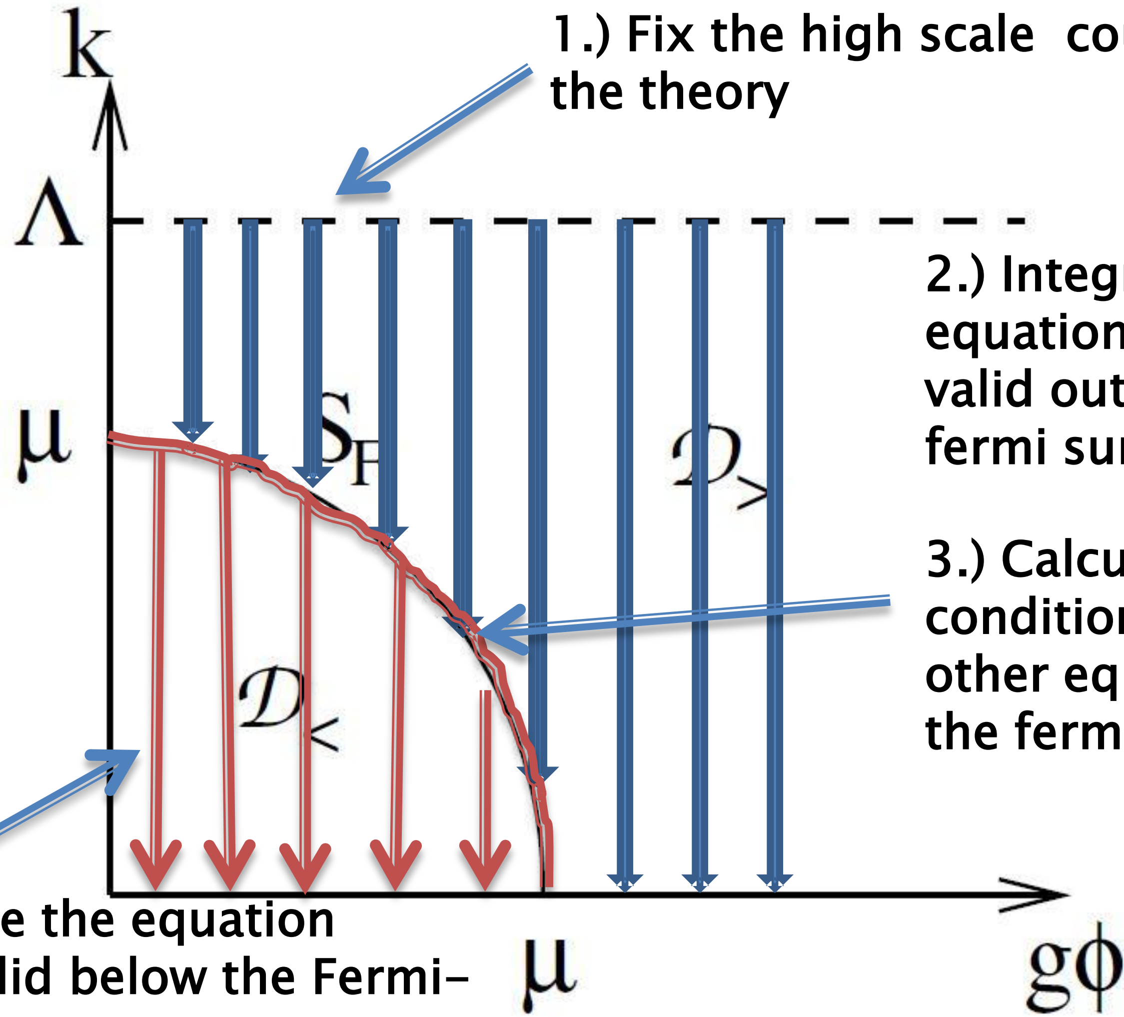
$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B}-\frac{4}{\omega_F}\right]$$

$k_F=\sqrt{\mu^2-g^2\varphi^2}$; Fermi-surface

$$\partial_k U_k = \frac{k^4}{12\pi^2}\frac{1}{\omega_B}$$

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel

Integration of the Wetterich-equaiton



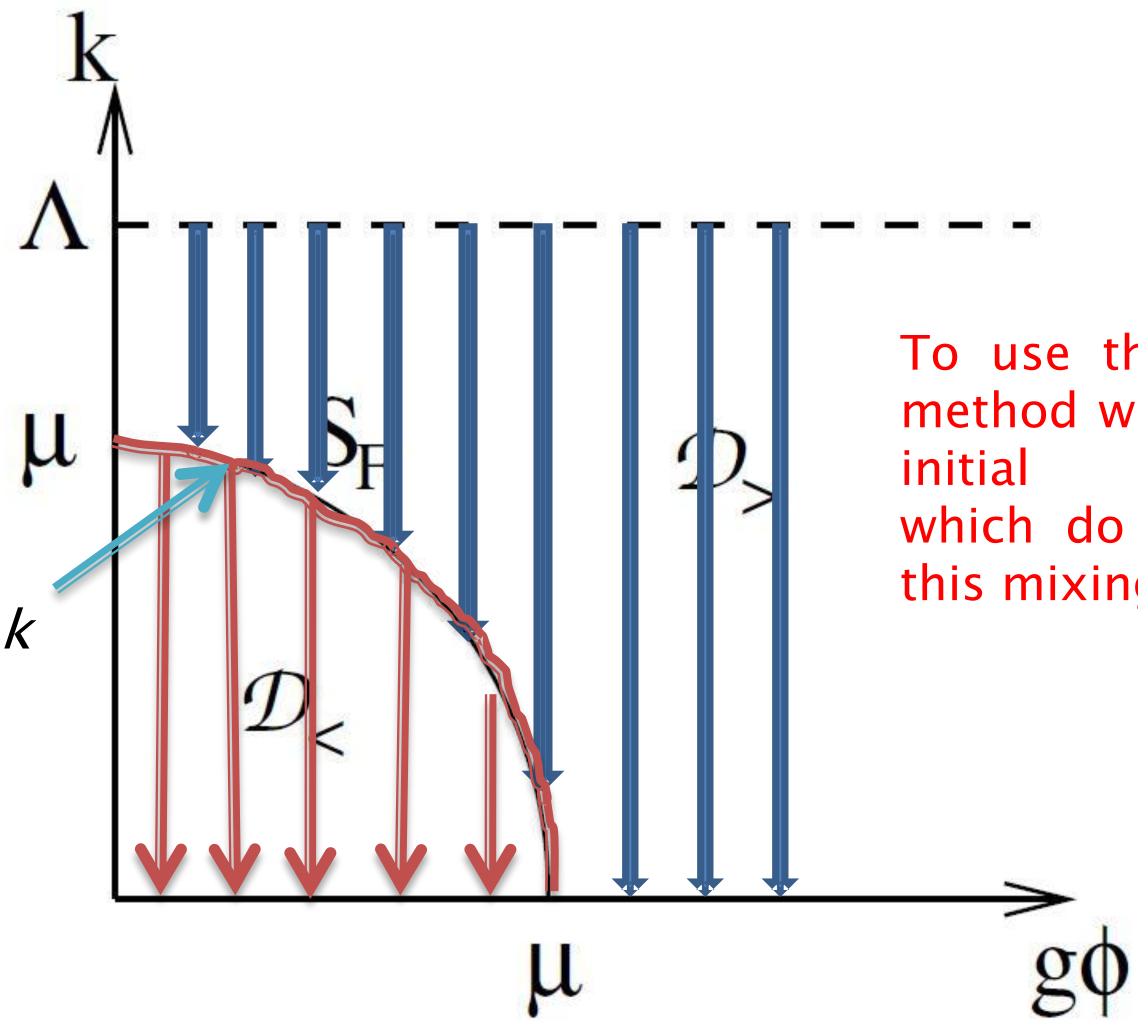
1.) Fix the high scale couplings in the theory

2.) Integrate the equation which is valid outside of the fermi surface

3.) Calculate the initial conditions for the other equation inside the fermi surface

4.) Integrate the equation which is valid below the Fermi-surface

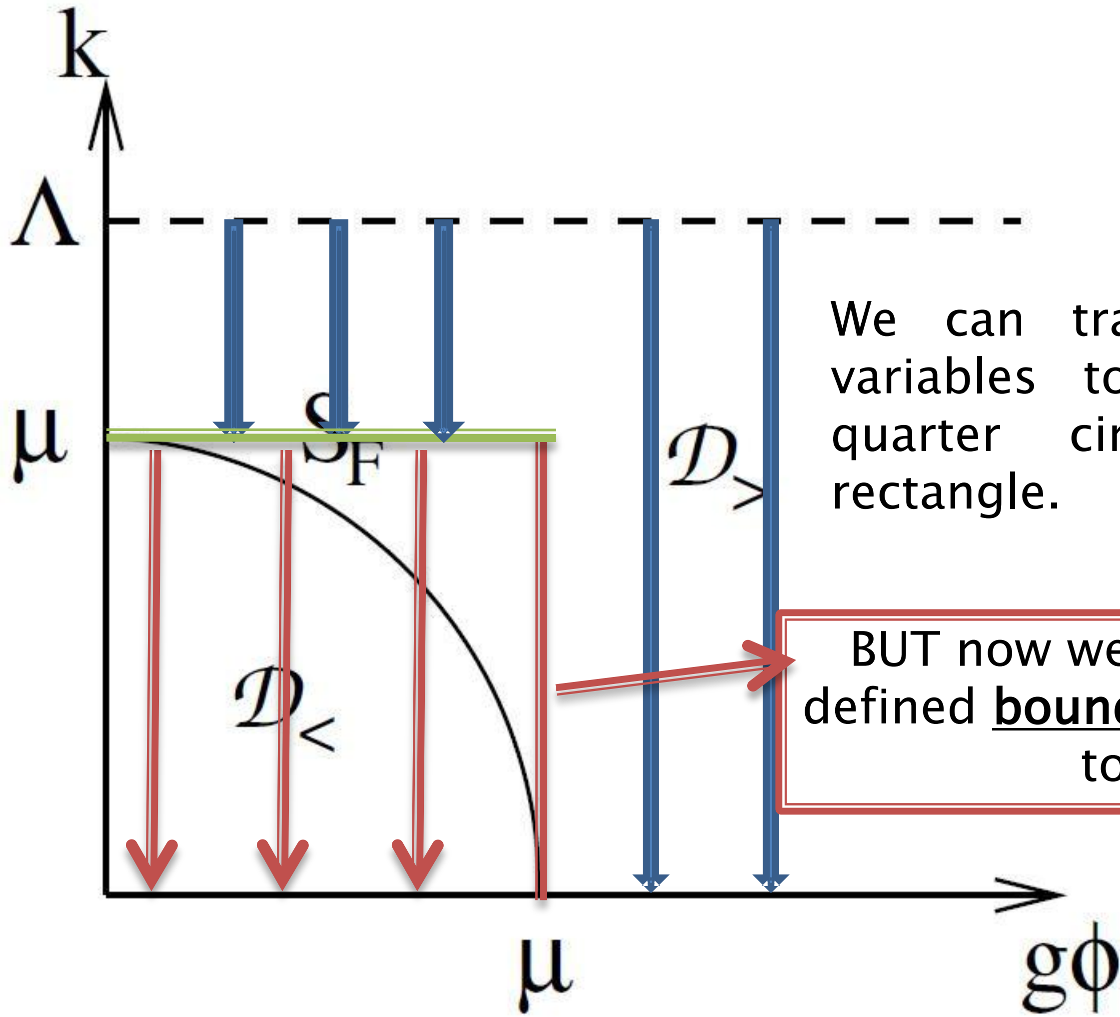
BUT…



The boundary condition mix *k* and $g\phi$

To use the orginal method we need an initial condition which do not have this mixing

Transform the variables



We can transform the variables to make the quarter cirle into a rectangle.

BUT now we have a well defined **<u>boundary</u>** **<u>condition</u>** too!

The transformed equation

- Circle-rectangle transformation: $(k,\varphi)\mapsto(x,y)\quad x=\varphi_F(k),\quad y=\frac{\varphi}{x}$
- Transformation of the potential: $\tilde{U}(x,y)=V_0(x)+\tilde{u}(x,y)$

Boundary condition at Fermi-surface

- The transformed Wetterich-equation:

$$x\partial_x\tilde{u}=-xV_0'+y\partial_y\tilde{u}-\frac{g^2(kx)^3}{12\pi^2}\frac{1}{\sqrt{(kx)^2+\partial_y^2\tilde{u}}}.$$

- And the new boundary conditions:

$$\tilde{u}(x=0,y)=\tilde{u}(x,y=\pm1)=0.$$

Solution by orthogonal system

- Solution is expanded in an **orthogonal basis** to accomodate the strict boundary conditoin in the trasformed area

$$\tilde{u}(x,y) = \sum_{n=0}^{\infty} c_n(x) h_n(y) \quad h_n(1) = 0 \quad \int_0^1 dy\, h_n(y) h_m(y) = \delta_{nm}$$

- The **square root** in the Wetterich-equation is also expanded:

$$x c_n'(x) = \int_0^1 dy\, h_n(y) \left[-x V_0' + y \partial_y \tilde{u} - \frac{g^2 (kx)^3}{12\pi^2} \sum_{p=0}^{\infty} \binom{-1/2}{p} \frac{(\partial_y^2 \tilde{u} - M^2)^p}{\omega^{2p+1}} \right]$$

Where: $\omega^2 = (kx)^2 + M^2$

Expanded square root

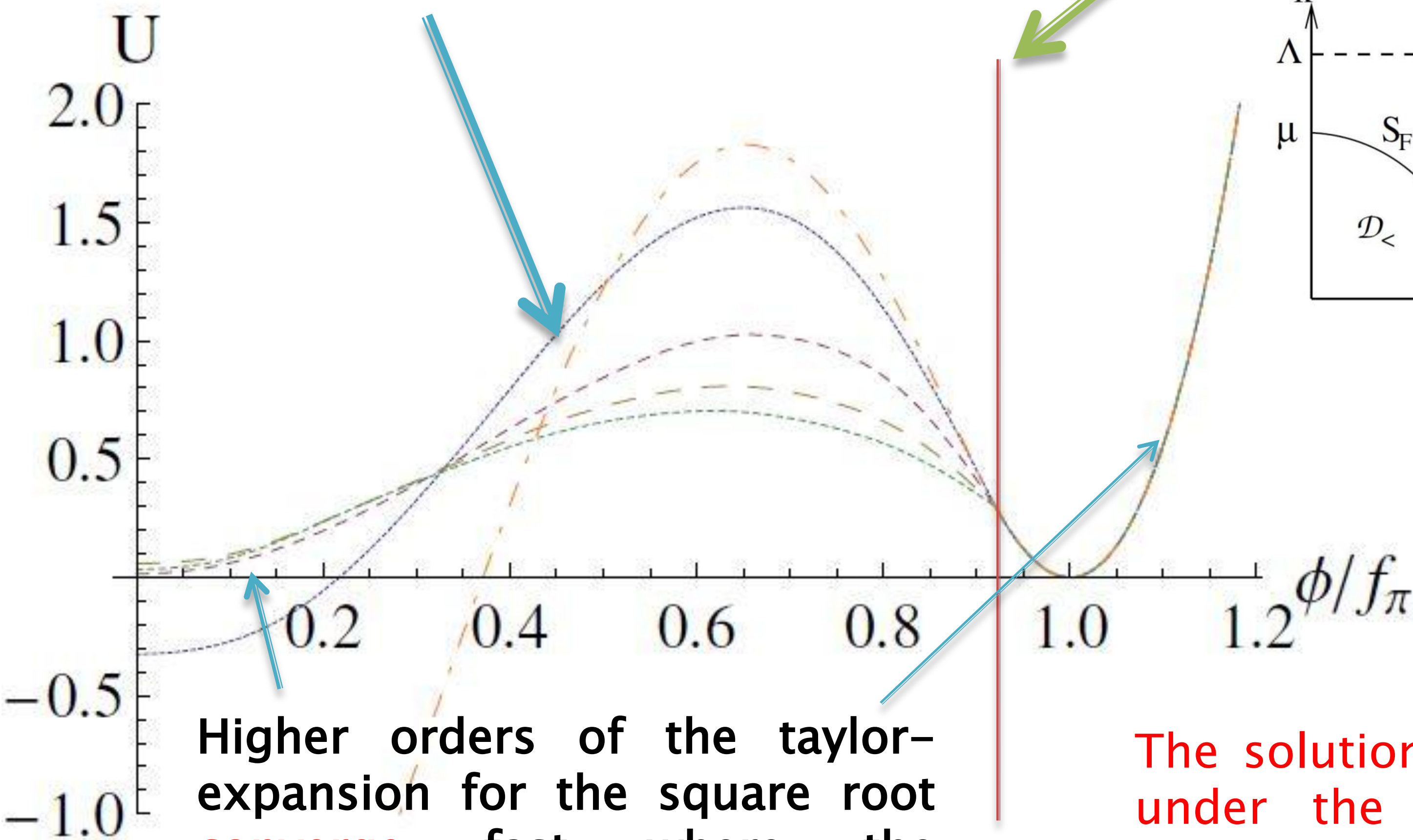
We use harmonic base

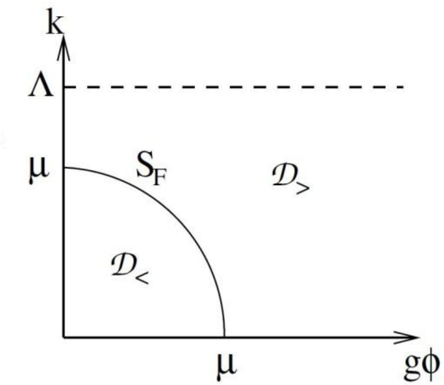
$$h_n(y) = \sqrt{2} \cos q_n y, \qquad q_n = (2n+1)\frac{\pi}{2}$$

Results-I

Potential in one-loop approximation

Fermi-surface in the field variable





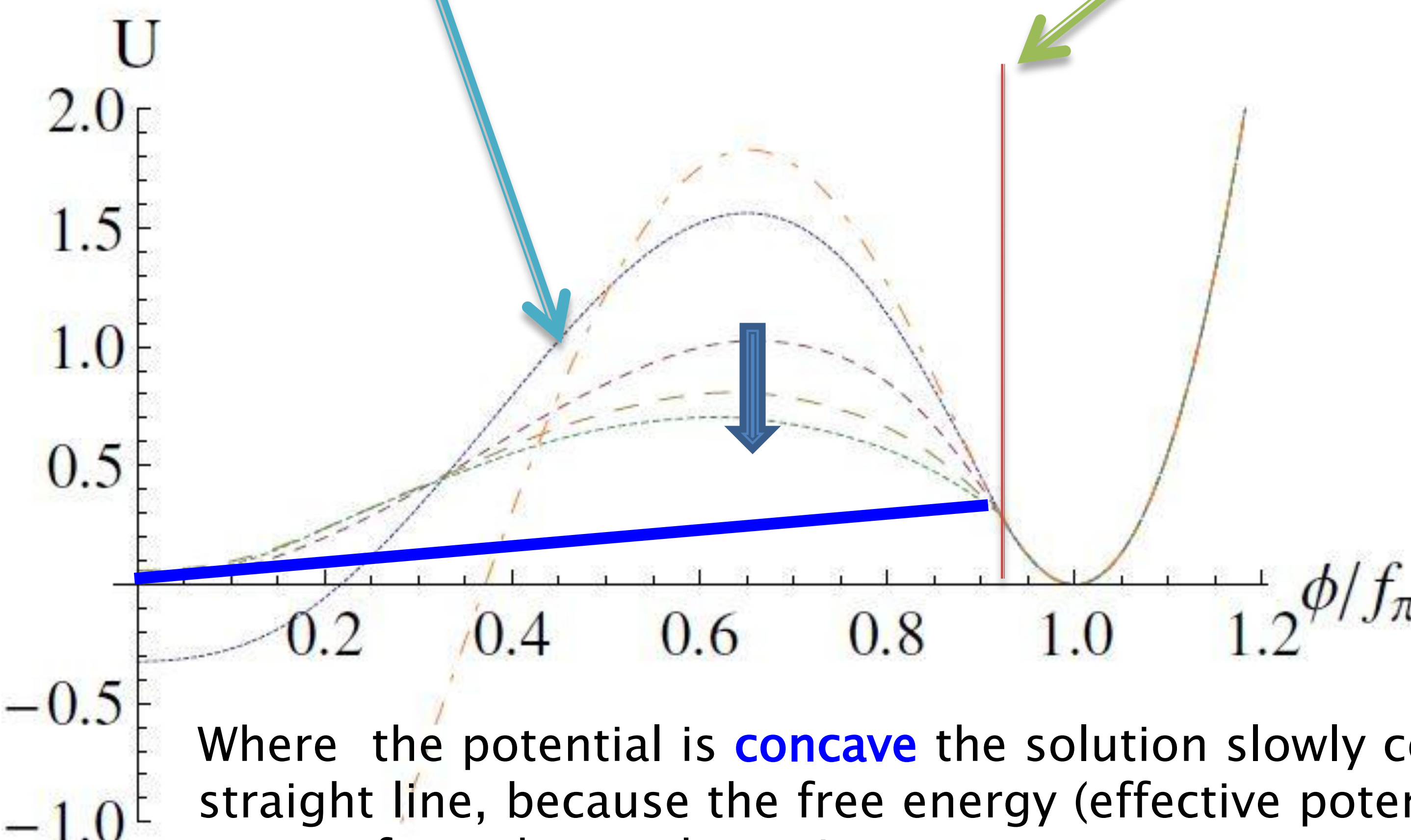
Higher orders of the taylor-expansion for the square root converge fast where the potential is convex

The solution changes only under the fermi-surface, because here we switch to the other equation

Results-I

Potential in one-loop approximation

Fermi-surface in the field variable

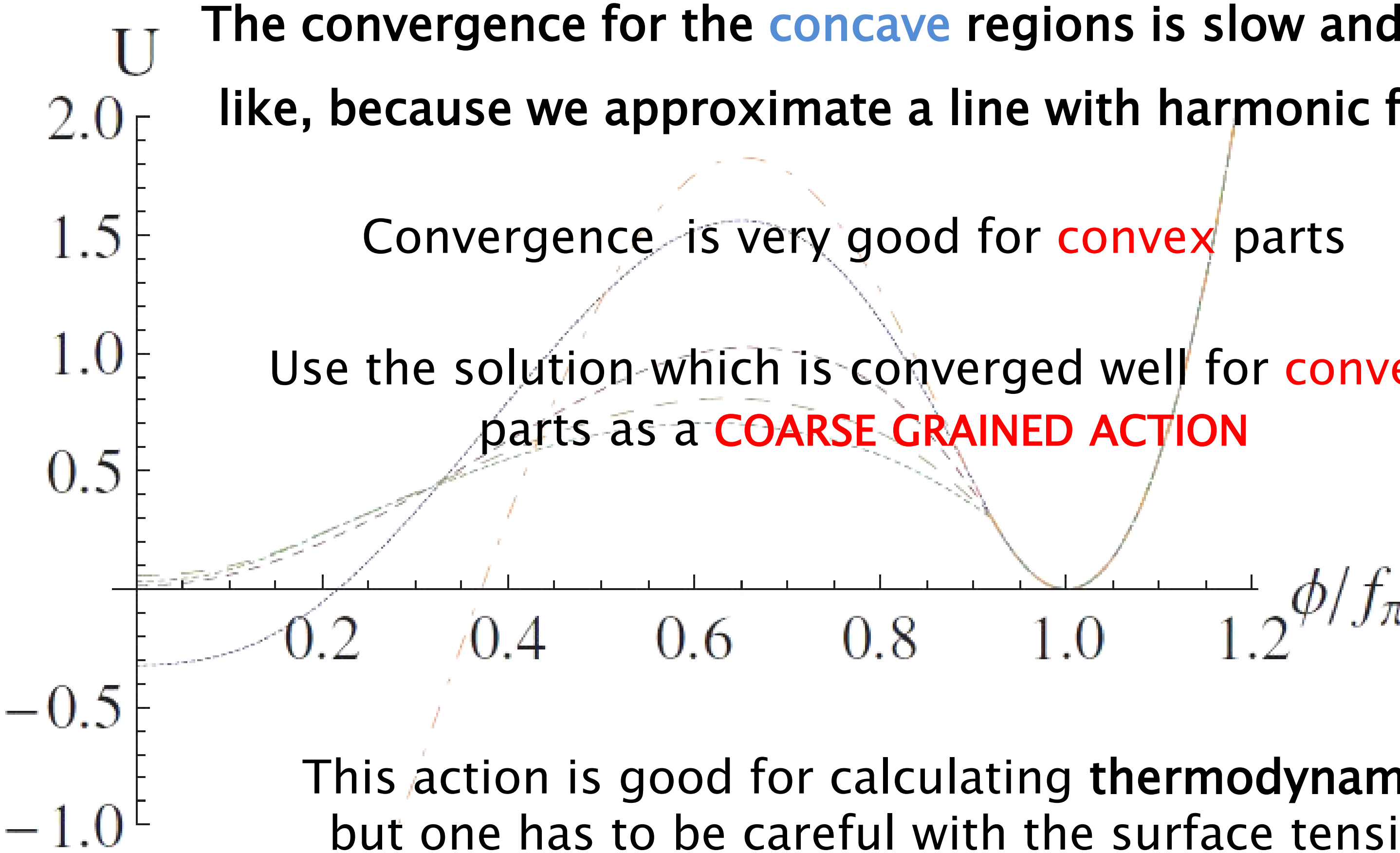


Where the potential is **concave** the solution slowly converges to a straight line, because the free energy (effective potential) must be convex from thermodynamics reasons.

This is the **Maxwell construction.**

Results-I

The convergence for the concave regions is slow and not point like, because we approximate a line with harmonic functions.



Convergence is very good for convex parts

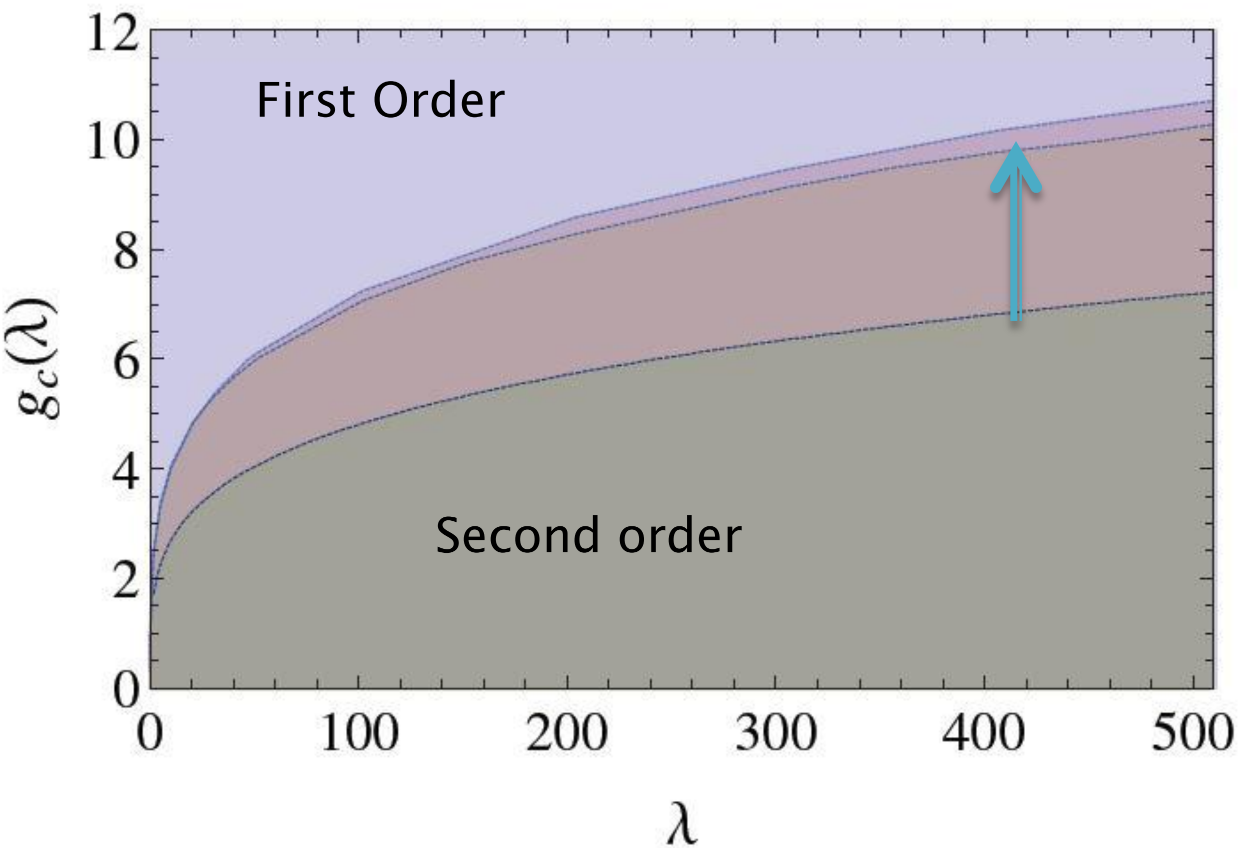
Use the solution which is converged well for convex parts as a COARSE GRAINED ACTION

This action is good for calculating **thermodynamics**, but one has to be careful with the surface tension

Results–II

Phase structure of the interacting Fermi-gas model

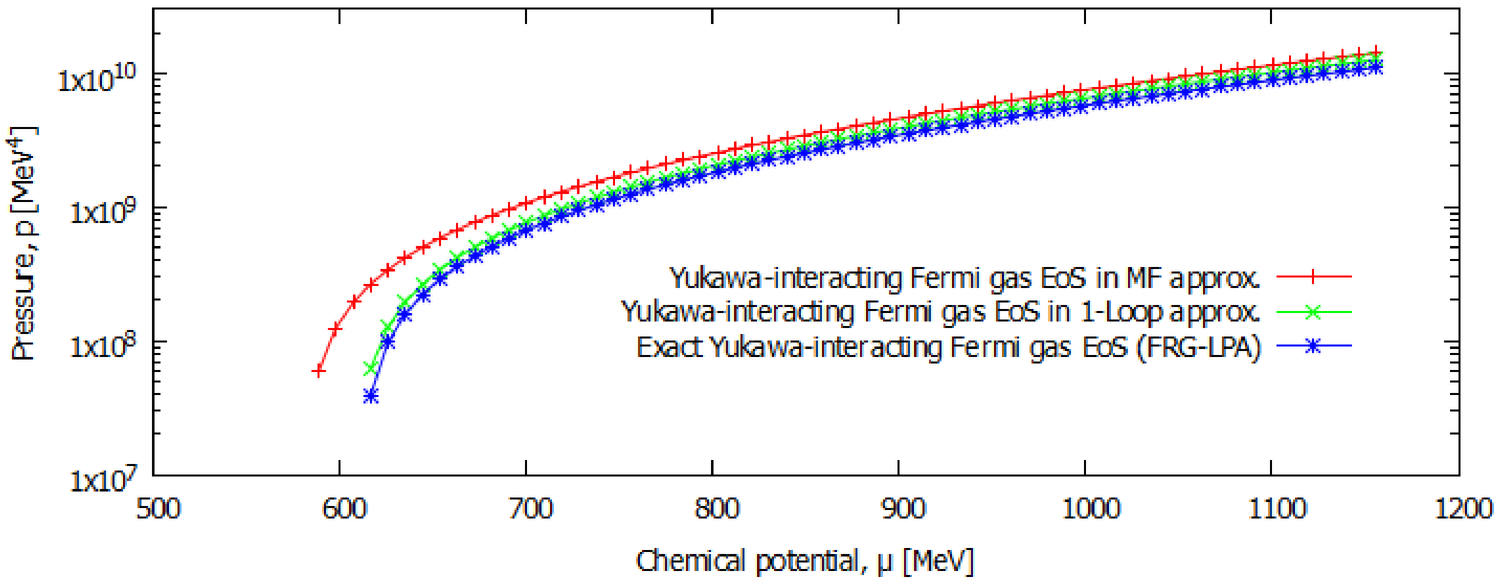
Yukawa coupling

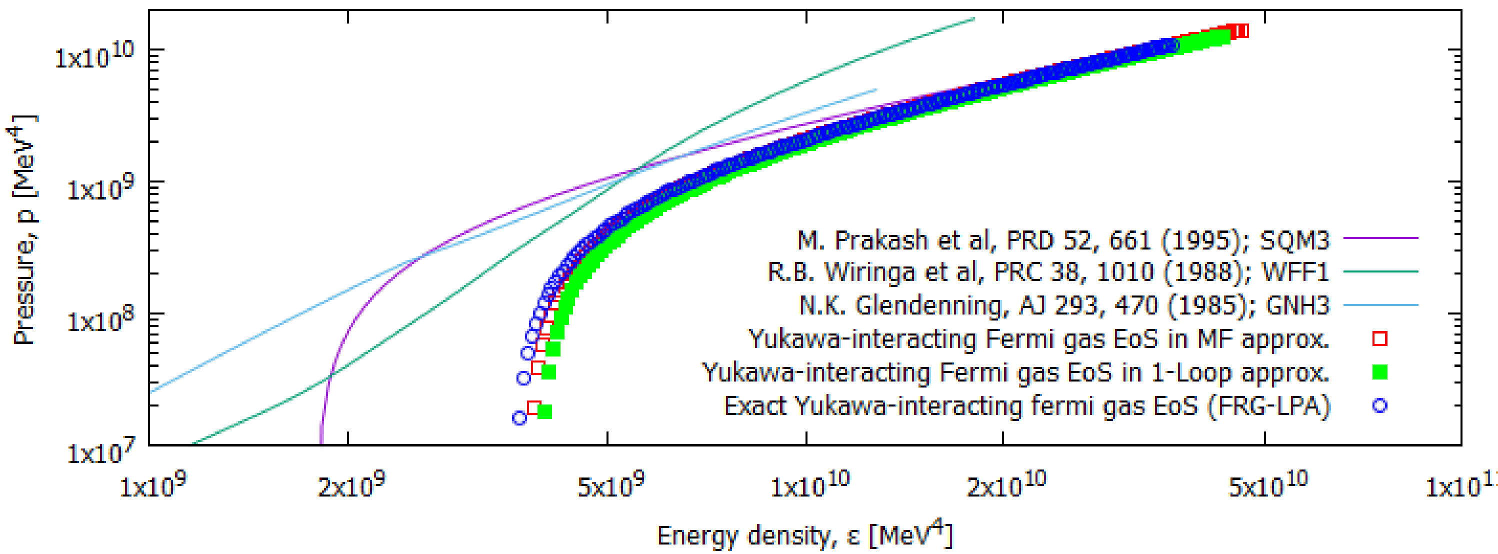


- FRG
- One loop
- Mean field

Scalar self interaction

The equation of state





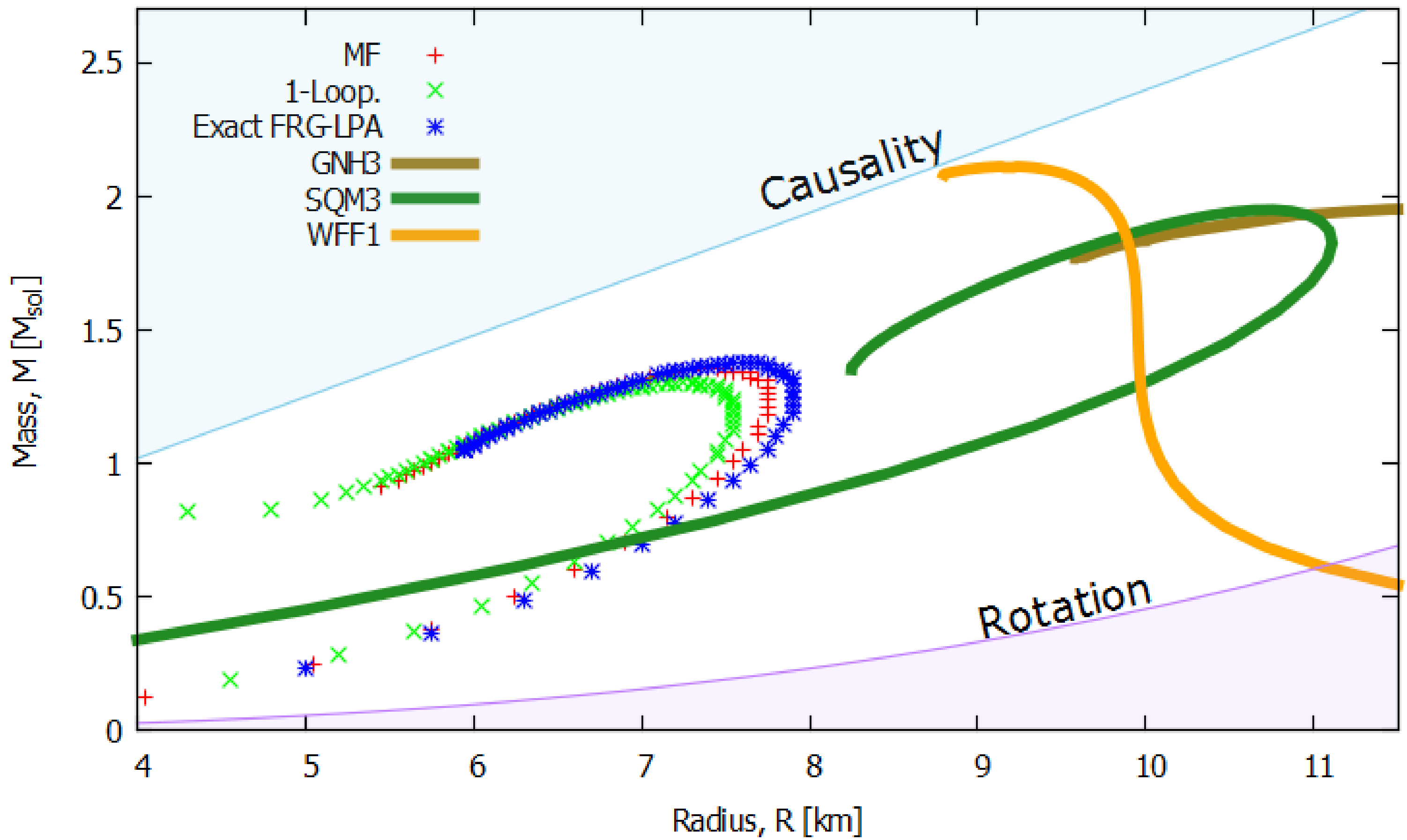
Application for compact stars

The Fermi-gas is not a realistic model of a neutron star, this is just demonstrates that a very small change in the EoS, means a noticeable change in the solution of the TOV equations.

$M_{FRG}=1.377$

$M_{MF}=1.358$

$M_{1L}=1.309$



Conclusions

- New method to calculate the running of the coupling constants
  - at zero temperature and finite chemical potential
  - Using Harmonic expansion to satisfy the boundary conditions
  - This is a general result: it can have other applications for Fermi fluids; for example in Condensed matter physics
- We demonstrated that
  - quantum fluctuations can have important role in effective models
  - They can have an effect on the properties of compact stars

Thank you for the attention !

Contact and related materials:
http://pospet.web.elte.hu/

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